Introduction to Optimization: Water Balloon Launch

University of Colorado Denver, GK-12: Transforming Experiences
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Goals:

- Expose students to optimization using an interdisciplinary focus of mathematics and physics
- Physics: Students will have a basic understanding of force and/or potential and kinetic energy.
- Mathematics: Students will be exposed to and have basic understanding of multivariable functions, mathematical models, curve fitting, and problem solving.
- Students will be able to effectively use experimental inquiry to design an experiment and control variables.
- Improve students' critical thinking and problem solving abilities.

Objectives:

- Students will be able to perform basic calculations used in optimization, such as objective functions and constraints.
- Students will be able to perform curve fitting tasks, multivariable functions, and model building.
- Students will be able to optimize some function beyond rudimentary guess-and-check
- Students will be able to use experiment to effectively utilize scientific inquiry to ask scientific questions, control variables, and analyze outcome data.

Standards

Science: Different parts of this lesson can be used to stress various science benchmarks. Problem-solving, experimental design, force, and energy are just a few.

Mathematics: This lesson can emphasize every standard for mathematics in Colorado.

- Develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems.
- Use algebraic methods to explore, model, and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.
- Use data collection and analysis, statistics, and probability in problem-solving situations and communicate the reasoning used in solving these problems.
- Use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.
- Use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.
- Link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.

Materials needed:

1. Launcher
2. Water balloons
3. Tape measure (at least 100 yards)
4. Launcher holder (must be a fixed and stable structure, ex: wood frame)
5. Rubber bands
6. Student lab book or notebook
7. TI Calculator with an overhead display screen. (For the Extend section)

(Minimal supplies needed: Launcher & tape measure)

Procedure:

Preparation:

- Build a launch holder (ex: wood frame) that is fixed and stable.

- Select location outside that is suitable for long distance water balloon launching

<u>Using the 5 E Instructional Model: A Framework for Inquiry-Based Instruction</u>

Engage:

(Day 1) Give every student a rubber band. Place a bucket in the middle of the classroom. Ask the students to try to shoot their rubber band into the basket. After most students have failed at the task, foster a discussion around why they think this has happened.

- Questions:
  - Why did this happen?
  - What do I already know about this to help me understand this better?
  - What have I discovered from this?
- Potential reasons for failure:
  - Others failed to adjust their shot after watching their peers
  - Rubber bands of different sizes or usage (loose vs. tight)
  - Pulling back the rubber band all the way is not always the best method
- Inform the students about the following day's goals and activities:
  - Fire some test water balloons and see how far they go
  - Use that data to fire a single water balloon to try to hit the GK12 Fellow or volunteer

Explore:

(Day 2) Provide each student (or group of students) with a water balloon. Each student must fill up his/her water balloon with water, weigh the mass, and record the information in his/her lab book. Take the whole class outside and measure the resting distance of the launcher from the launch holder. (For consistency, the handles of the launcher must be fixed so that the only variable is how far back the student wants to pull the launcher.) Then, have each student/group of students decide a distance of how far back they want to pull the launcher and predict the distance the balloon will travel. As each student/group launches their water balloon, have a designated pair of students hold the tape measure (held at a designated spot on the launch holder) and announce the distance traveled of each water balloon launched. Each student/group will record the following data for every water balloon launched on their data sheet:

- distance launcher was pulled back
- mass of water balloon
- distance traveled

Once everyone has fired the launcher and recorded the data, return to the inside classroom to review.

Explain:

To help students formulate big ideas and clarify misunderstandings, ask some of the following questions:

- What were your predictions?
- Which water balloon did you think would travel the farthest?
- Why do you think you got the results you did?
- Did you run into any problems?
- What were your variables?

Elaborate:

After students have analyzed the data and come up with an explanation(s), provide the class with 1 balloon with a mass of X. Tell the class that the Fellow/volunteer will stand Y distance away from the launcher and that the class must decide as a whole how far back to pull the launcher in order to hit the fellow/volunteer with the balloon. Y should be a distance between values they've already achieved, but not yet recorded. Remind the students that they only get one chance to hit the fellow/volunteer. Take the students outside and let them launch the balloon to see if they can hit the target.

Bring the students back inside to compare and analyze. Students will input the data into the TI calculator to graph the data, and plot the mass vs. distance curve.

***E**valuate*:

- Formal
    - Collect students' lab books
    - Pose additional questions for students to answer on their own:
        - Such as: Prescribe a firing position (distance launcher is pulled back) and ask the students to vary the mass of their balloons to see what mass goes furthest. Then, have the students graph the data, fit a curve, and discover the vertex of the parabola to find the optimal mass for the given pull-back distance.
    - Ask a real-life question that uses similar problem solving and optimization techniques:
        - Such as: You operate a business that produces Wii's, Playstation's, and Xbox's and the profit earned per console is $10, $20, and $30 respectively. (Provide constraints) Each console takes X amount of time for production and X amount of material. Ask the students how many of console they should make to make the most money. Look for results beyond blind guess and check. (Most students will invent sophisticated and unique ways to find the optimal solution.) Ask students to make a graph of their results.

References:

- Trowbridge, L. W., Bybee, R. W., & Powell, J. C. (2000). Teaching secondary school science: Strategies for developing scientific literacy.