[Title for Lesson] Designing an Electricity Transportation Network

- **[Resident Name] Jeffrey Larson**

[Date of Preparation] April 23, 2010
[Middle School- Grade Level] Englewood Leadership Academy, Grades 6-9
[Type of Lesson] Mathematics Lesson
[Date of Implementation] May 5-6, 2010
[Approximate Time Length] 1.5 hours, but it can easily be extended/contracted.

OVERVIEW

[Abstract/Short description: topic focus, activities, and purpose] The students will attempt to design an optimal transportation network for electricity.

PURPOSE

[Why is this lesson important?] There is currently an online contest, which is the inspiration for this lesson, with 2000 Euros of prize money awarded to the best networks. This will give students an opportunity to work on real-world problems, and see the underlying concepts of these difficult problems are within their reach. (The important part of the lesson isn't figuring out the cost of a proposed network by hand(though that is an essential skill). The goal is to use math to figure out the best network design.)

OBJECTIVES & STANDARDS MET

[Objective: Students will be able to…] design a network to connect power plants to cities, while meeting each city's respective needs.

[List all state and national standards met. Include the # of the standard.]
All of the “Six Goals for Colorado Students of Mathematics” are met by this lesson:

1. **Become mathematical problem solvers: The students are solving a real-world problem**
2. **Learn to communicate mathematically: In their write-ups, the students will have to convey their solution, as well as the reasoning that went into it.**
3. **Learn to reason mathematically: Every student will have to think rationally in order to design a network to meet every city's demand.**
4. **Make mathematical connections: This lesson connects previous topics about energy use, geometry of lines, and many others.**
5. **Become confident in their mathematical abilities: Every student can find an answer to this problem (though not optimal), which should empower every student for future problems.**
6. **Learn the value of mathematics: Everyone needs electricity in their city, and math can help deliver it.**

Also, the lesson meets the following state standards:
Standard 2: Using algebraic methods to explore, model, and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.
Standard 4: Solve problems and model real-world situations using geometric concepts. Specifically, grades 5-8 calls for the ability to “Solve problems using coordinate geometry.”
Standard 5: Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.
Standard 6: Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.

Reference: http://www.cde.state.co.us/cdeassess/documents/OSA/standards/math.htm#standards

BACKGROUND INFORMATION & REFERENCES

- **[Background: What must students already be able to do before this lesson? What concepts have to be mastered in advance to accomplish the lesson objectives?] We will provide a short introduction to electricity for all students, which will ensure everyone has a basic understanding. as well as how to calculate distances between cities. They will need to be able to compute basic unit conversions (if 1 km of power line costs 300,000 Euros, how much does it cost to run a straight line from a power plant to a city 75.6 km away?).**

[References: List references that were used to prepare your lesson plan. If you modify an existing lesson plan,

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cite that source. Verify ALL information – you are the math/science expert in the classroom] I am using the current contest from the French "Fédération des Jeux Mathématiques" website. The problem statement is attached to this document, and can also be found at: http://www.scmsa.com/ for the time being.

VOCABULARY, MATERIALS, PREPARATION, SAFETY

- **[List Lesson Vocabulary] We will define "electricity" and "power plant" as a quick review. Any technical term in the following sentence will be defined, but in a simplistic way: "A transformer is needed to move between Very High Tension line (which has a capacity of 400,000 Volts), and High Tension line (which has a capacity of 90,000 Volts).**
- **[Materials Required: List all materials required to complete the lesson] The students will need paper and pencil, as well as something to make calculations.**

[Preparation: Detailed description of the preparation that is needed before the lesson and during the class period] If the students have a chance to read the contest description before class, that would greatly reduce the preparation time. Other than that the lesson should be self contained.

[Saftey: List any safety concerns that will be reviewed with students prior to lesson implementation] N/A

METHOD: 5 E'S MODEL

Describe the step-by-step procedures for each E of the 5 E's model:

[Engage] The students will be inherently interested in a large cash prize, so in my limited experience, there shouldn't be a problem "engaging" the students. I will introduce the problem to the students, especially highlighting the cash to the best answer. For those students who may not be interested in the cash, we will discuss how essential electricity is to the modern world, as well as the advantages of low-cost solutions to many similar problems. These concerns should highlight the problem's importance.

[Explore] I will lead a discussion of what constitutes a "good" electricity network, ensuring the students bring up the following constraints:

- **Each city must be connected to the network and have its demand met.**
- **The network provides enough redundancy so any one network component's failure doesn't prevent cities from receiving electricity.**
- **There are certain protected areas (National Parks, natural landforms, etc) which cannot have Very High Tension line running through it.**

We will also discuss the cost of constructing a network (since connecting every city to every power plant would be sufficiently redundant, but also ridiculously expensive). If the students don't discover this concern on their own, I will draw two different networks on the board (one with few lines and one with many line) and ask which network is better.

[Explain] I will then delve into the rules for the contest, show the students the map of cities/power plants, and explain the minimum requirements for a feasible network, which are bulleted above. (An example of a city/power plant layout with various demands/supplies can be found in the attached pdf, but any map can work.)
I'll talk about the cost for different network components (cost per kilometer for different lines, cost of transformers), and explain that the winner is the network that meets all energy requirements with the minimum cost. (Again, costs are highlighted in the attached pdf, but any reasonable costs will work.)

[Elaborate] I will highly suggest students search for a feasible network first, and I will have students present their ideas for constructing a feasible network to the class as they arise. We will then discuss methods for reducing the cost of a feasible network. (Such one idea may be splitting lines into/out of the cities and plants.)

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[Evaluate] Each student will be required to hand in a feasible network, as well as state its cost. Each will have to write/type up a summary of their network design, state how much it costs, and explain their thought process while designing. We will be having the students stop regularly throughout the lesson to jot down any progress.

ADAPTATIONS OR DIFFERENTIATED LEARNING

- **[Include all potential adaptations that may be useful for the classroom.] This lesson is for a variety of students, so discretion should be taken when defining the problem. If 15 cities and 6 power plants are too many for the students, the number of cities/power plants can be reduced to a more manageable amount. The only reason I am using these numbers is to encourage the students to work on an actual problem from the real-world, not some approximation to the problem. Also, for 6th graders, I might ignore the transformers or discussion of different types of power lines.**

EXTENSIONS & CONNECTIONS

- **[List possible extensions for gifted students or further explorations. List possible connections to other subjects.] Students can put as much time into this lesson as desired. I am certain I will have students work on this at home. With the various entry/exit points, gifted students can explore countless possibilities of network design.**

HANDOUTS & PRESENTATIONS

- **[Attach all handouts, powerpoints, overheads, worksheets, etc. to the end of this document]**

The student evaluation form and the contest description is in the zip file along with this lesson.

PEER REVIEW COMMENTS

[List any peer comments that aided in the preparation of this lesson]

REFLECTIONS (COMPLETED AFTER LESSON IS IMPLEMENTED)

[How does this lesson integrate your research into the classroom?]
In general, the concept of working on problems without known solutions is essential to research. Specifically, this problem is centered around difficult optimization problems; finding their solution is what my research entails. Of course, I wasn't able to introduce the students to high-level techniques for solving these problems, but I think every student thought the problem was tractable.

[How could the lesson be improved?]
If I had a computer program that allowed the students to instantly design a network by clicking to connect cities, it would have enabled them to quickly check many networks. Though I considered writing a computer program, the overhead to make a GUI was more than I had time for. Without a way to quickly check many networks, students weren't able to see how many networks they could have designed, and how truly difficult it is to find

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the best network.

[What worked well?]
Have two arrangements of cities/plants let the students choose which problem they wanted to work on. This let students work on the problem they found acceptable.

[What did you learn?] I may have made the problem too easy, or at least appear so. I wanted to show that middle school students could understand and work on real-world problems that confound current mathematicians. In stressing how easy it was to find a feasible network, I didn't stress enough how difficult it is to find the optimal network. Especially after reading the Day 1 Forms, I saw many students say the problem was "easy" which helped me on Day 2.

[How does this impact your future profession?] I hope to introduce people to such problems as appropriate. I already planned on doing so, but seeing the success only reinforced my belief that students can grasp the fundamentals of difficult problems.

[Further thoughts...]

STUDENT WORK EXAMPLES (COMPLETED AFTER LESSON IS IMPLEMENTED)

[Include examples of student work from this lesson - includes worksheets, tests, pictures, etc.]

Designing an Electricity Transportation Network

a game organized jointly by :

The French "Fédération Française des Jeux Mathématiques"

and the

Société de Calcul Mathématique SA

with the support of

RTE
(Réseau de Transport d'Electricité)

Total prizes amount: 2,000 Euros

Description of the game

One has to design a transportation network for electricity, very high voltage and high voltage, taking into account the usual requirements:

- each city must be connected to the network ;
- the network provides enough redundancy, so that a break of some component does not prevent any user from receiving electricity ;
- the network respects protected zones.

There are 2,000 Euros for prizes: 1,000 for individual answers and 1,000 for collective answers (schools) ; see below.

Opening date for the game: April 1st, 2010.

Closing date for the game: June 30th, 2010. All answers must be received by that date.

All answers must be sent by email, in .pdf format, to the French "Fédération Française des Jeux Mathématiques". Email address : ffjm@wanadoo.fr

Languages for the answers: English or French.

The principle of the game is simple: you are given six electricity plants, characterized by their position and power, and fifteen cities, characterized by their position and consumption. You have to build a distribution network, connecting the plants to the cities. The winner is the one who builds the best network, that is the network with smallest installation cost.

All constructions and presentations must be explicit, and their quality will be an important factor in the selection of candidates.

I. The game

1. The country

It will be represented schematically by a square of 1,000 km for each side. The origin of axes will be the point down left, and the two axes will be the corresponding sides of the square. The unity is the km.

2. General data

Here are the six power plants, with their coordinates and production:

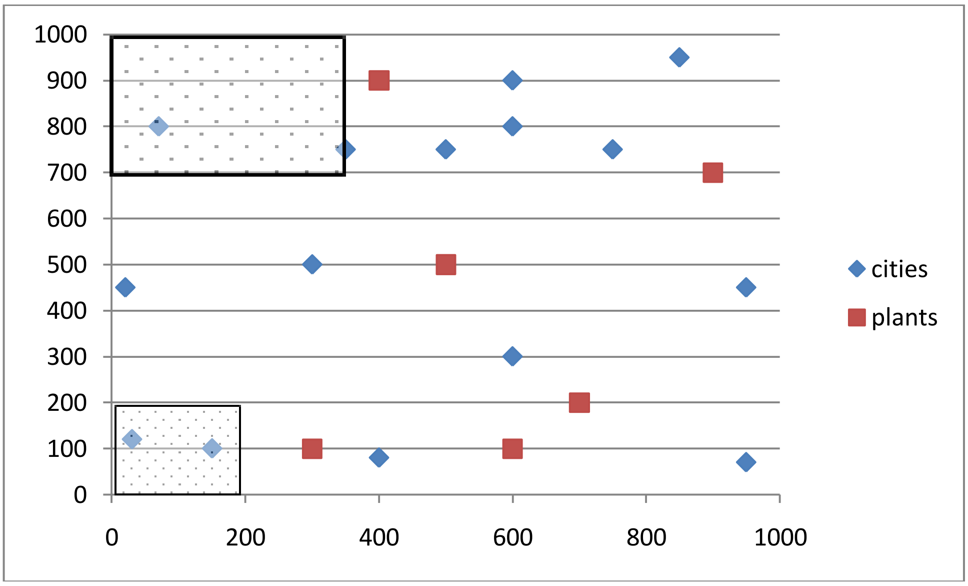
Plants	x	y	production (MW)
C1	300	100	900
C2	600	100	500
C3	700	200	1 200
C4	900	700	450
C5	500	500	750
C6	400	900	1 200
total			5 000

Table 1: the plants

Here are the cities, with their coordinates and consumption:

City	x	y	consumption (MW)
V1	150	100	200
V2	400	80	300
V3	950	70	200
V4	30	120	250
V5	600	300	300
V6	20	450	250
V7	300	500	300
V8	950	450	300
V9	70	800	250
V10	350	750	150
V11	500	750	250
V12	600	800	300
V13	600	900	100
V14	750	750	250
V15	850	950	250
total			3 650

Table 2: the cities



Map of the country, with the cities, the plants and the protected zones

3. Objective

The objective is as follows: each city must receive High Tension electricity. A city is considered as a point: the alimentation of houses inside the city is made with low voltage, and this does not concern the present problem, which deals with high tension only.

4. Electrical Lines

Two kinds of lines exist: Very High Tension (VHT), namely 400,000 Volts, and High Tension (HT), namely 90,000 Volts. The plants always produce VHT. The amount of electricity which is lost during transportation, due to Joule effect, is lower when tension is higher, so one always has to use VHT lines whenever possible, even though the installation cost is higher.

The cities must receive HT current; so a conversion from VHT to HT is necessary. It is done by transformers. The number and position of these transformers is part of the problem. The only constraints about them are:

- They cannot be installed within 1 km from each city ;
- They cannot be installed in a protected zone, defined below.

The objective of the game is to install the VHT and HT lines and the transformers, in order to bring electricity to all cities, at minimal cost.

5. Data

We give:

- Cost for each km, VHT line : 1 million Euros
- Cost for each km, HT line : 300 000 Euros
- Cost for each transformer : 500 000 Euros

6. Protected zones

Two zones are considered as protected, because the habitants do not want to see electricity lines. They are:

$0 \le x \le 200$, $0 \le y \le 200$ (left bottom corner)

$0 \le x \le 350$, $700 \le y \le 1000$ (top left corner).

In these zones, one cannot put VHT lines; one must use underground HT lines, and their cost per km is 1 million Euros.

7. Constraints

They are as follows:

- Each city must receive its electricity ;
- Any VHT or HT line cannot carry more than 1 GW ;
- Any underground HT line cannot carry more than 0.3 GW ;
- The network must work even if one of its components is subject to failure. This means that all cities must be served, even if one VHT line breaks down, or one HT line, or one transformer.

Important: we do not ask that the network brings electricity to everyone in case of two components breaking down at the same time.

II. Comments

Electricity production and consumption are supposed constant over time; we do not take into account daily or monthly changes (this, of course, does not reflect reality).

The distance we consider is the Euclidean direct distance.

We do not take into account the losses in electricity due to transportation, or due to the transformers.

The objective of the game is to minimize the installation cost; we do not care about the exploitation cost. This cost has been taken into account, when we said that all lines must be VHT, whenever possible.

Each line may be divided, as often as one wants; the same line may connect several plants and/or several cities.

There may be crossings between lines, with or without connection.

Several lines may be put close to each other, in parallel, but this does not reduce the costs.

III. Prizes to be given by FFJM and SCM, with support from RTE

There are two categories:

Individual Prizes:

For the winner : 500 Euros
For the second : 200 Euros
For each of the next three : 100 Euros each.

Collective Prizes (schools or academic institutions):

For the winner : 500 Euros
For the second : 200 Euros
For each of the next three : 100 Euros each.

The best solutions will be published on the web sites of FFJM, SCM and RTE.

The official announcement of the results, and prize ceremony, will happen on Thursday August 26th, 2010, in Paris, for the final of the Championship of Mathematical and Logical Games, "Maison Internationale de la Cité Internationale Universitaire de Paris", 17 Boulevard Jourdan, 75014 Paris, France.

The general rules of the game are available on the web site of FFJM. Participating in the game implies that you accept these conditions.

Student Name: ____________________
Date:_____________________

To build my network would cost:____

My network is feasible because:

When designing my network, I tried to make it as cheap as possible by doing the following:

Day 1

Student Name: ______________________________

Grade:______________________________

Class:______________________________

I think this problem is really___________________ because:

I have the following ideas for making a feasible network:

Tomorrow, I am going to try to:

Day 2

Student Name: ______________________________

Grade:______________________________

Class:______________________________

I am trying to solve the following problem:

The hard part of this problem is:

The easy part of the problem is:

FINAL FORM

Student Name: ______________________________

Grade:______________________________

Class:______________________________

To build my network would cost:____

My network is feasible because:

When designing my network, I tried to make it as cheap as possible by doing the following:

If I had more time, I would do the following to make my network even cheaper:

I problem this lesson was really ____________________ because:

Solving these types of problems are important because: