# Beads, Balls, and Beakers- A Sphere Packing Exploration

Matt Nabity and Elizabeth Untiedt
Interdisciplinary Lesson
Middle School, Grade 8
Duration: Two 90-minute blocks

Implementation: May 3-7, 2010, Falcon Bluffs Middle School, Littleton, CO
Lesson Prepared: April 2, 2010

Overview
In this lesson, students will be introduced to the concept of molecular packing, a phenomenon which economizes the volume of space occupied by complimentary molecules. Students will explore this concept on a macro scale by experimenting with packing bouncy balls, gumballs, and fuzzy pom-poms into beakers.

Purpose
This lesson covers a critical component of the 8th-grade physical science curriculum: molecular interactions. The intensive use of formulas and area calculations reinforces the supporting role of mathematics in scientific study. In addition, the uncertain measurements and the propagation of uncertainty through the formulas demonstrates Matt's research in interval uncertainty, and the difference between the firm balls and the fuzzy balls demonstrates Elizabeth's work with fuzzy set theory.

Objectives
At the end of this lesson, students will be able to explain the space saving effect of combining molecules of complimentary diameters. They will display competence in calculating volumes of spheres. In addition, they will understand that there are such things as interval and fuzzy uncertainty, and will understand that uncertainty increases when calculations are performed on uncertain quantities.

Standards Met
**CO State Science:**
Standard 1 (Scientific Investigation Process): Benchmarks 2 (measurement/gathering data), 3 (interpreting data), 5 (identifying alternative explanations); Standard 2 (Matter and Energy): Benchmarks 3 (conservation of mass), 5 (atomic structure), 6 (compound/molecular structure); Standard 5 (Knowledge Building Process): Benchmarks 4 (predictive models)
**National Science:**
Standard 1 (Properties and Changes of Properties in Matter)
**CO State Mathematics:** Standard 4 (Geometric Concepts in Problem Solving): Benchmark 5 (Volume in 3 dimensions); Standard 5 (Measurements in Problem Solving): Benchmarks 1 and 2 (Using measurements of volume and mass; Using measurements to make comparisons)
**National Mathematics**:
Standard 1 (Problem Solving): Indicator 3 (build new knowledge through problem solving); Standard 4 (Mathematical Connections): Indicator 2 (apply math in contexts outside of math) and 3 (interconnection of mathematical ideas); Standard 5 (Mathematical Representation): Indicator 1 (modeling physical phenomena)

Background: Students should be able to accurately measure lengths and masses. Students should be able to understand how to cube a quantity when it appears in an equation. Students should be familiar with the theory of conservation of mass, basic atomic structure, and how compounds/mixtures are formed. Students should be able to square and cube quantities.

References: N/A

Lesson Vocabulary: sphere, diameter, pi, mass, molecule, molecular diameter, fuzzy, interval, uncertainty, precision, apparent volume, real volume, characteristic property

Materials required
Per class:
- 100 ml ethanol
- 150 ml water
- 2 250 ml beakers

Per group of students (recommended group size is 3 or 4 students):
- 100 1cm marbles;
- 8 1 inch (27mm) bouncy balls;
- 8 1 inch (27 mm) pom-poms (fuzzy balls)
- 1 caliper, 3 rulers or other device for measuring diameters of balls and beakers
- Triple-beam balance or other device for measuring mass of balls and beakers

Preparation: Fill one bin per group with 3 rulers, 2 beakers, 8 bouncy balls, 100 beads on Day 1.
Fill one bin per group with 3 rulers 2 beakers, 12 pompoms, and 100 beads on Day 2.

Safety: Students will not handle ethanol. Basic lab safety procedures will be followed and students will be reminded that a beaker full of bouncy balls can, when dropped, create mayhem.

Method: 5 E's Model

**Day 1**

Engage: This is a demonstration in the front of the classroom. One beaker will be filled with 100 ml of ethanol, and one beaker with 150 ml of water, for a total of 250 ml of total liquid. However, when the water is added to the ethanol, the resultant will be less than 250 ml in volume. (Approximate amount will be added to final version of the lesson plan.)

Ask students to discuss in group why this might happen. (sample answer: volume is not a characteristic property.)

Have students record a hypothesis about what is going on in their handouts.

(This phenomenon occurs because the water molecules fill the space between the ethanol molecules efficiently, but the students will come to the conclusion through the following explore and explain steps.)

This part of the lesson should take 10-15 minutes.

Explore: Students will recreate the water/ethanol phenomenon on a macro level with bouncy balls and beads.
- -Fill one beaker to the 150 ml line with bouncy balls (this will take two layers of four balls each).
- -Fill the other beaker to the 100 ml line with glass beads (approx. number TBD.)
- -Pour the beads into the bouncy balls, agitating gently so that the beads fill the spaces between.
- -Measure the apparent volume of the mixture.
- -It will not be 250 ml. Have students hypothesize why. Give them a chance to make a new hypothesis about the water/ethanol mixture.

This part of the lesson should take about 30 minutes.

Explain: Students will discover the difference between the apparent volume and the actual volume by doing measurements and calculations.

-Measure diameters and masses of beads, balls, and beakers, as required by the attached worksheet.

This includes uncertainty. Matt will introduce propagation of uncertainty.

This part of the lesson will take about 40 minutes.

**Day 2**

Explain, continued:

-Calculate real and apparent volumes.

This includes uncertainty.

-See that mass and real (non-air) volume is conserved, but apparent volume (which includes air) is not.

-Explain molecular phenomenon using you-tube video and/or by massing ethanol and water.

This part of the lesson will take about 30 minutes.

Elaborate: Students will repeat the explore and explain steps from above (minus the mass), replacing bouncy balls with pom-poms.

Students will hypothesize why pom-poms produce different results from the bouncy balls.

Elizabeth will introduce fuzzy set theory.

Students will update hypothesis to reflect fuzzy set theory.

Evaluate: Students will produce a paragraph explaining the water-ethanol phenomenon, with an accompanying illustration.

Students will write a paragraph describing the role of uncertainty in applied math.

Adaptations or differentiated learning

[Include all potential adaptations that may be useful for the classroom.]

Extensions & connections

Finite precision. Molecular Packing.

Peer review comments

[List any peer comments that aided in the preparation of this lesson]

Reflections (completed after lesson is implemented]

[How does this lesson integrate your research into the classroom?]

[How could the lesson be improved?]

[What worked well?]

[What did you learn?]

[How does this impact your future profession?]

[Further thoughts...]

student work examples (completed after lesson is implemented]
[Include examples of student work from this lesson - includes worksheets, tests, pictures, etc.]

# Beads, Balls, and Beakers- A Sphere Packing Exploration

Where math and science meet…

## Demonstration

What is the volume of water?

What is the volume of ethanol?

What do you think will happen when they are combined?

What *did* happen when they were combined?

Make a hypothesis about what happened:

## Exploration

Fill one beaker to the 150 ml line with bouncy balls.

How many balls did you use?

Fill the other beaker to the 100 ml line with glass beads.

How many beads do you think you used?

Pour the beads into the bouncy balls, agitating gently.

What is the apparent volume of the mixture?

Is this the same as 100 ml + 150 ml?

What do you think is happening?

Now, what do you think happened with the water and the ethanol?

## Data Collection

| | Mass | Mass Uncertainty | Number | Diameter | Diameter Uncertainty | Height | Height Uncertainty |
|---|---|---|---|---|---|---|---|
| ( units ) | ( g ) | ( +/- g ) | ( balls ) | ( cm ) | ( +/- cm ) | ( cm ) | ( +/- cm ) |
| Beaker | | | | | | | |
| Bouncy ball | | | | | | | |
| Bead | | | | | | | |

Leading questions for Matt's discussion of uncertainty

## Calculations

| | Radius = Half of diameter | Volume Formula | Calculated Volume per unit | Calculated Volume Uncertainty per unit | Total Actual Volume | Total Actual Uncertainty | Total Mass |
|---|---|---|---|---|---|---|---|
| ( units ) | | | | ( +/- cm ) | | | |
| Beaker | | $V = \pi \cdot r^2 \cdot h$ | | | | | |
| Bouncy ball | | $V = \frac{4}{3} \cdot \pi \cdot r^2$ | | | | | |
| Bead | | $V = \frac{4}{3} \cdot \pi \cdot r^2$ | | | | | |

## Interpretation

What is the mass of the bouncy balls?

What is the mass of the glass beads?

What is the mass of the bouncy balls and glass beads combined?

Is mass conserved?

What is the apparent volume of the bouncy balls?

What is the apparent volume of the glass beads?

What is the apparent volume of the bouncy balls and glass beads combined?

Is apparent volume conserved exactly?

What is the actual volume of the bouncy balls?

What is the actual volume of the glass beads?

What is the actual volume of the bouncy balls and glass beads combined?

Is actual volume conserved exactly?

What is the uncertainty in the actual volume of the bouncy balls?

What is the uncertainty in the actual volume of the glass beads?

What is the uncertainty in the actual volume of the bouncy balls and glass beads combined?

Is actual volume conserved if you take uncertainty into account?

## Show your understanding

What causes the difference between actual and apparent volume?

How did uncertainty affect the conclusion of this experiment?

# Getting Fuzzy- A Sphere Packing Expansion

## Exploration

| |
|---|
| Fill one beaker to the 150 ml line with pompoms. |
| How many pompoms did you use? |
| Fill the other beaker to the 100 ml line with glass beads. |
| How many beads do you think you used? |
| Pour the beads into the pompoms, agitating gently. |
| What is the apparent volume of the mixture? |

## Data Collection

| | Number | Diameter | Diameter Uncertainty | Height | Height Uncertainty |
|---|---|---|---|---|---|
| ( units ) | ( balls ) | ( cm ) | ( +/- cm ) | ( cm ) | ( +/- cm ) |
| Beaker | | | | | |
| Pompom | | | | | |
| Bead | | | | | |

Leading questions for Elizabeth's discussion of fuzzy uncertainty

## Calculations

| | Radius = Half of diameter | Volume Formula | Calculated Volume per unit | Calculated Volume Uncertainty per unit | Total Actual Volume | Total Actual Uncertainty | Total Mass |
|---|---|---|---|---|---|---|---|
| ( units ) | | | | ( +/- cm ) | | | |
| Beaker | | $V = \pi \cdot r^2 \cdot h$ | | | | | |
| Pompom | | $V = \frac{4}{3} \cdot \pi \cdot r^2$ | | | | | |
| Bead | | $V = \frac{4}{3} \cdot \pi \cdot r^2$ | | | | | |

## Interpretation

What is the apparent volume of the pompoms?

What is the apparent volume of the glass beads?

What is the apparent volume of the pompoms and glass beads combined?

Is apparent volume conserved exactly?

What is the actual volume of the pompoms?

What is the actual volume of the glass beads?

What is the actual volume of the pompoms and glass beads combined?

Is actual volume conserved exactly?

What is the uncertainty in the actual volume of the pompoms?

What is the uncertainty in the actual volume of the glass beads?

What is the uncertainty in the actual volume of the pompoms and glass beads combined?

Is actual volume conserved if you take uncertainty into account?

## Show your understanding

Was the difference between actual and apparent volume more or less than in the first part of the experiment?

How did fuzziness affect the outcome of this experiment?

Draw a picture and write a paragraph to explain what happens when water and ethanol and mixed *and explain why.*