Fair Division of Cookies

Breeann Tonnsen   **Creighton Middle School**
Content   **November 6, 2009**

OBJECTIVE, BACKGROUND INFORMATION, & REFERENCES

Objectives:
The primary objective of this lesson is to introduce ideas of fair division and teach one method that is used in this practice. Secondarily, the objective to reinforce ideas about fractions and area.

Standards:
1.4: use the relationships among fractions, decimals, and percents, include the concepts of ratio and proportion, in problem-solving situations
5.1: estimate, use, and describe measures of distance, perimeter, area, volume, capacity, weight, mass, and angle comparison;
5.2 estimate, make, and use direct and indirect measurements to describe and make comparisons;

Background:
Teachers will need to know some fair division methods. Students will need to know how to estimate area using a transparency that has small until squares. Student will also need to know some basic arithmetic including division, fractions, and addition.

Resources:
Information from Mike Ferrara from a GK-12 meeting.
Classroom materials from **Game Theory** taught in summer 2008 by Jason Williford.
University of Alabama website: http://www.ctl.ua.edu/math103/FairDiv/continuo.htm

VOCABULARY, MATERIALS, PREPARATION, SAFETY

Vocabulary:
1. Fair Division
2. The Last Diminisher Method
3. Area
4. Divider-Chooser method
5. Equal parts of a whole

Materials Required:
- Worksheets created by me
- Unit squares copied onto transparencies
- Smart board for prompt and fair division instruction
- Cookies and treats for students with diabetes
- Utensils for cutting cookies

Preparation:
To be prepared for the lesson, we will have to have purchased the materials listed above. Plus, we will do a mini-lesson the week before showing them how to estimate area with the transparencies.

Safety:
There is a safety concern about the utensil used to divide up the cookies. However, we will purchase soft cookies and use a utensil that is neither sharp nor serrated.

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Engage:
We will engage the students by having the smart board say: "We are dividing up cookies to eat today!" as their warmup for the day.

Explore:
The students will explore fair division in three parts.
1. First students will find a worksheet on their desk with a picture of a cookie. The prompt on the smart board will be to spend 5 minutes discussing in their groups how they are going to divide up the 'cookie' in front of them fairly without using measuring devices. They will need to answer the following questions on their worksheet: Does one person get the cookie? Or do multiple people get the cut the cookie? How did you decide? Who gets to pick the first piece? Why? How will your group know if the cookie is divided up fairly?
2. Then students will be given a minute or two to actually use their method to divide the cookie amongst the group members.
3. Then, we will pass out a transparency with the unit square on them. They will be told to estimate the area of each part of the cookie as a group. Then they will be asked to answer the following questions on the worksheet: About how big was each part of the cookie? Was it divided into equal pieces? What would you do differently next time (if anything)?

Explain:
After the students have finished exploring, I will bring the whole class into a guided discussion about the different methods. This discussion will emphasize the idea of how you determine if the division is fair or not. Hopefully one group of two will have thought about the divider-chooser method, where one person cuts and the other person chooses first. Then I will tell them that fair division is a subject studied by mathematician. Then I will explain the Last Diminisher method to them.

The Last Diminisher Method: *A generalization of the Divider-Chooser Method for N players*

1. Randomly order Players: $P_1 , P_2 , \dots , P_N$.
2. P_1 cuts a piece.
3. P_2 either claims a subpiece of P_1's piece (becomes a Diminisher) or passes to stay in contention for a piece of the rest.
4. Case 1: If P_2 *becomes a Diminisher:*
   The difference between the 2 claims is put back with the rest and P_1 becomes a Contender.
   Case 2: If P_2 *passes:* Play passes to P_3.
5. Each remaining player, in turn, diminishes or passes. When all have played, the last diminisher gets his/her piece and departs.
6. The process begins again with one less player. When 2 players remain, use Divider-Chooser.

Elaborate:
We will put a large oddly shaped cookie on the smart board. We will have one or two students come up to the front of the board. The students and I will use the Last Diminisher Method on the smart board cookie. Then, we will hand out cookies to the groups and have the students divide them up using either their original method of the Last Diminisher Method.

Evaluate:
Students will hand in their worksheets at the end of the period for evaluation on effort. Then the next day that I am at the school, I will give them a warmup that asks questions about the lesson. For example, "when might this idea of fair division be used in real life?" and "Why is the word fair used and not equal?"